Teaching Practice of Advanced Mathematics Based on PBL Teaching Method

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Abstract. Advanced mathematics is an important foundational course for science and engineering students, with strong abstraction, logic, and application. However, the existing teaching of higher mathematics shows a characteristic of emphasizing knowledge teaching and neglecting ability and literacy teaching. Facing the problems in higher mathematics teaching mentioned above, this article first proposes a six-step teaching model based on the PBL teaching method. This is a teaching model that follows the six steps of "problem introduction, knowledge construction, independent exploration, group demonstration, summary and improvement, and practical expansion". Secondly, the six-step teaching model is applied to the teaching practice of higher mathematics. Taking the teaching of Lagrange multiplier method as an example, the teaching process of the six-step teaching model is elaborated. Through the classroom design process of raising questions, communicating and discussing, and solving problems, students' learning initiative is cultivated and their thinking ability is exercised. Teaching practice has shown that the six-step teaching method is very helpful for cultivating students' abilities and training innovative thinking.

Keywords: PBL teaching method; Lagrange multiplier method; Six-step teaching mode

1. Introduction

Advanced mathematics is a basic course for engineering students, which plays an important role in cultivating students' mathematical knowledge, creative ability and mathematical literacies. For engineering students, on the one hand, the knowledge of advanced mathematics has a profound influence on the study of subsequent courses of engineering majors. On the other hand, learning the thought method in advanced mathematics is helpful to cultivate students' thinking ability. Under the background of China's "new engineering" construction and professional certification, higher requirements are put forward for students' innovative ability and practical ability.

However, advanced mathematics teachers are mathematics professionals with a single professional background and lack engineering knowledge and practice. Therefore, there are insufficient teaching resources such as engineering cases that can be used for advanced mathematics teaching, which cannot support the requirements for cultivating students' engineering practical abilities. Due to factors such as a large amount of course content and tight academic hours, there is a phenomenon of "emphasizing knowledge impartation and neglecting ability cultivation" in advanced mathematics teaching. Although students acquire a lot of concepts, theorems, and other knowledge, their subjective initiative in learning mathematics is insufficient, and they lack thinking, which can easily lead to knowledge forgetting. At the same time, in the course teaching process, students lack practical opportunities to apply mathematical knowledge to solve practical problems, resulting in poor improvement of their innovation and practical abilities. Therefore, advanced mathematics teaching requires new resources and methods for teaching reform to adapt to the new requirements of the new era.

The PBL (problem based learning) teaching method was first proposed by American education and psychologist Jerome Bruner in the 1950s. At the end of the 20th century, the research and teaching practice of PBL teaching method began in the field of education in China, especially in the field of medicine. This article mainly studies how to use the PBL teaching method to carry out teaching activities in advanced mathematics, elaborates on the implementation process of the PBL teaching method, and demonstrates how to guide students to analyze problems through the proposal

of practical problems. In the process of solving problems, new problems are set up, linked together, and corresponding mathematical concepts and theorems are proposed. In this process, students' learning ability, ability to analyze and solve problems, and innovation ability are cultivated.

2. The Practice of PBL Teaching Method in Advanced Mathematics Teaching

2.1 Construction of curriculum resources

Advanced mathematics teachers should aim at students' ability to apply mathematical knowledge to solve engineering problems and their subsequent professional development needs, construct advanced mathematics curriculum resources that combine with engineering background knowledge, and cultivate innovative talents with solid mathematical theoretical foundations and outstanding mathematical modeling abilities for engineering problems. Course resources should include the following aspects.

Firstly, literature materials containing mathematical knowledge sources

In order to break the teaching model of "definition theorem example practice" in reproducing knowledge, and enable students to realize that the content system supporting advanced mathematics is not just a castle in the air, and that every concept originates from the solution of practical engineering problems, teachers should organize the concepts and practical sources of definition in advanced mathematics, and form a literature database that showcases the sources of mathematical knowledge for course teaching, To cultivate students' practical and innovative engineering abilities.

Secondly, an application case library based on engineering background

In order to enhance students' abilities in knowledge application and mathematical modeling, and provide support for the construction of new engineering subjects, advanced mathematics teachers should extract and transform complex engineering problems into application cases that are suitable for advanced mathematics teaching and extracurricular exploration and discussion. They should develop application cases with engineering backgrounds and apply them in teaching practice to enhance students' mathematical modeling and mathematical knowledge application abilities.

Thirdly, the "step by step" mathematical modeling practice projects

In order to enhance students' ability to apply mathematical knowledge and innovate, advanced mathematics teachers should set up a "foundation improvement challenge" hierarchical mathematical modeling practice project based on the teaching content, combined with students' professionalism and learning situation. Basic level projects mainly include projects that use mathematical software for simple calculations or drawing, to enhance students' mathematical software operation ability. The improvement project requires students to establish mathematical models and analyze simple practical problems, in order to enhance their ability to analyze and model practical problems. The challenge level project requires students to establish mathematical models and solve complex engineering problems to enhance their application and innovation abilities.

Fourthly, the "Foundation & Exploration & Practice" layered course exercises

In order to balance the differences in basic levels, individual differences, and development needs among students, and to focus on improving students' abilities and cultivating their qualities, advanced mathematics teachers can establish an objective question bank for the learning foundation, which is used for frontline learning detection, classroom practice, and real-time testing; Paper homework books for calculation and comprehensive questions, used for after-school exercises to consolidate basic knowledge; Improve and expand the question bank (mainly focusing on postgraduate entrance exams and competition questions), laying the foundation for students to meet higher level challenges; Topic discussion question bank, guiding students to actively think and explore independently; Group collaboration application question bank to enhance students' practical application abilities.

2.2 Innovative Teaching Mode Based on PBL Teaching Method

Teachers adhere to the teaching philosophy of "student-centered development", take classroom teaching as the main body, and use online learning platforms to construct a six-link teaching model of "problem introduction, knowledge construction, independent exploration, group demonstration, summary and improvement, and practical expansion"(see Fig.1). They adopt problem-oriented heuristic teaching, and conduct classroom activities such as case analysis, teacher lectures, classroom

quizzes, group collaboration, and communication demonstrations, realize the re-creation of knowledge, enhance scientific thinking and innovation ability.

2.2.1 Question Introduction: Teachers raise questions through thematic discussions, mathematical case studies, engineering case sharing, and other methods, and provide step-by-step questions for more complex problems to stimulate students' thinking; Release pre class task sheets to guide students to use online learning platforms' video and literature resources to independently complete their learning and understand the practical sources of knowledge generation.

2.2.2 Knowledge Construction: Guide students to construct knowledge through methods such as exploring cognitive roots, transferring knowledge analogies, and recreating knowledge. For concept teaching, mathematical case studies based on "tracing the root cause" guide students to start from the root problem of mathematical concepts, personally experience the process of their generation, and achieve the re-creation of knowledge; For the teaching of theorems and methods, knowledge transfer is generated through knowledge analogies, including low-dimensional to high-dimensional, finite to infinite, and horizontal knowledge analogies, in order to acquire new knowledge. In the process of knowledge re-creation, students actively explore, boldly guess, and dare to innovate, which is a training of their innovation ability

2.2.3 Self-exploration: Adopting a problem driven approach to inspire students to actively analyze and independently solve problems. Teachers carefully screen and design questions to guide students to review old knowledge and learn new knowledge in simple and diverse forms; Students actively seek solutions to problems, or propose new problems, discuss with each other, interact with teachers, and gradually develop the habit of thinking independently. In this way, it not only stimulates students' enthusiasm and interest in learning, helps them deepen their understanding of abstract theory, but also stimulates their innovative spirit of daring to question and explore in the process of continuous thinking and bold exploration of problem-solving.

2.2.4 Group Presentation: Communicate and discuss the results of independent exploration in groups, question and affirm each other, form the final result, and present it to enhance students' sense of achievement and honor, and cultivate their teamwork, critical and questioning spirit.

2.2.5 Summary and Improvement: Based on communication and presentation, organize and summarize knowledge, enhance understanding and application abilities of knowledge, experience the mathematical ideas contained in knowledge, and cultivate students' ability to summarize and summarize.

2.2.6 Practice Expansion: After class, hierarchical tasks include: graded question bank exercises, data reading, case analysis, practical tasks, etc. Practice expansion is carried out through science thinking training bases, utilizing practical links such as thinking training projects and mathematical modeling projects to enhance students' innovative practical abilities and cultivate the spirit of scientific exploration.

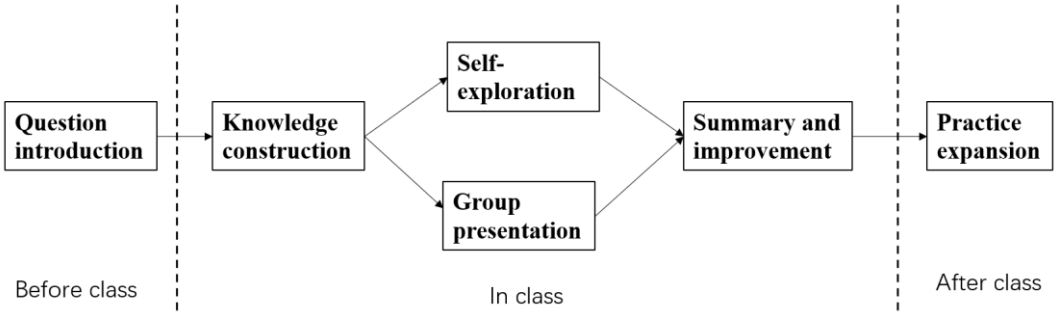

Figure 1. The six-step teaching model

3. Teaching Practice of Lagrange Multiplier Method Based on PBL

The conditional extreme value problem is a common extreme value problem in daily life and engineering applications. The Lagrange multiplier method is an important method for solving conditional extremum problems. The construction of this method is very clever and the expression is

extremely concise. What is the thinking of mathematicians behind this beautiful formula? The teaching design of this lesson will start from the practical problem of selecting a post location, and reproduce the creative process of the Lagrange multiplier method through a combination of geometric observation and theoretical deduction, and apply it to the solution of practical conditional extreme value problems.

3.1 Question Introduction: Before class, teachers use online learning platforms to release autonomous learning tasks and guide students to think about what is a conditional extremum problem? What is the difference between conditional extremum and unconditional extremum? And set up some practical questions for students to choose which ones belong to conditional extremum problems.

By asking questions, students can master the concept of conditional extremum problems and clarify the connotation and extension of conditional extremum problems.

After completing these basic pre-class tasks, students work in groups to find conditional extremum problems in daily life or engineering, and establish their objective functions and constraints. Cultivate students' ability to discover problems, abstract thinking, and mathematical modeling.

3.2 Knowledge Construction: In class, the teacher summarizes the conditional extreme value problems in life and engineering found by classmates, and obtain a general expression of the conditional extreme value problem. Then introduce the issue of campsite selection:

Assuming a new checkpoint has been built on one side of a border defense road, in order to facilitate communication between the checkpoint and the outside world, it is now decided to address and build a camp along one side of the road. Where can the camp be built so that the straight-line distance between it and the checkpoint is the shortest?

Setting question: Is this problem a conditional extremum problem? What are the objective functions and constraints? How to solve it?

Under the guidance of these problems, the students may establish the mathematics model of this problem:

$$\min d^2 = x^2 + y^2$$
$$s.t., \varphi(x, y) = 0$$

Where $\varphi(x, y) = 0$ represents the road.

3.3 Self-exploration: Students first solve the problem of camp site selection, and obtain the solution method through graphic demonstration. The site of the camp should to be the tangent point of the curve $\varphi(x, y) = 0$ and $x^2 + y^2 = C$. That is the tangent point of the conditional curve and one contour line of the objective function.

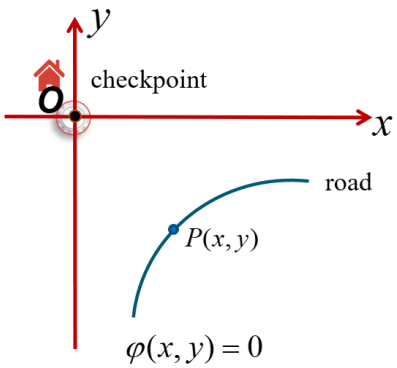

Figure 2. The diagram of the checkpoint and the road

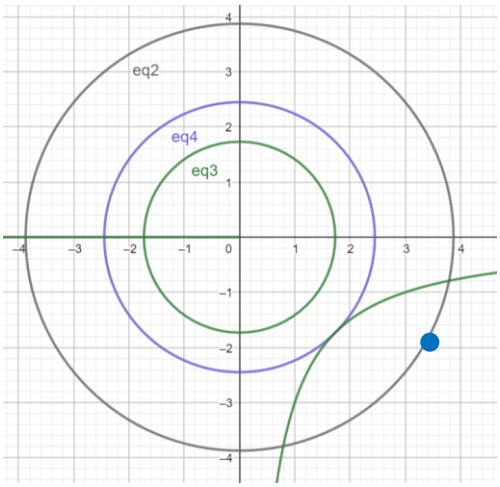

Figure 3. Solution of the camp site selection problem

3.4 Group Presentation: The teacher askes: Is the solution to the campsite selection problem applicable to general conditional extreme value problems? Then the teacher guides students to work in groups to theoretically derive the necessary conditions for conditional extremum points and write the correct expression of the Lagrange multiplier method.

3.5 Summary and Improvement: Classmates summarize the conditions and conclusion of the Lagrange multiplier method, and to think how we can do if there is more than one condition of the extreme value problem. After discussing, the students may get the Lagrange multiplier method for two conditions extreme value problems.

3.6 Practice Extension: Establish a mathematical model and solve the conditional extreme value problems found by each group in actual life or engineering practice for pre class tasks.

4. Conclusions

Traditional teaching methods have certain advantages in establishing knowledge architecture and theoretical derivation. PBL teaching method, as an emerging teaching method based on constructivism, advocates for students to explore independently and cooperate with each other to acquire knowledge, which is consistent with the goal of cultivating students' innovative and practical abilities in the context of the construction of "New Engineering". In order to meet the talent needs of the construction of "New Engineering", promote the teaching reform of advanced mathematics by combining PBL teaching method with traditional teaching methods. Practice has shown that this approach can not only lay a solid theoretical foundation for students, but also cultivate their innovative and practical abilities, enhance their awareness of teamwork, and further improve the teaching quality of advanced mathematics

Acknowledgements

This work is supported by Xi'an Technological University teaching reform research projects (20JGY45 and 21JGG001).

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