Student-Centered and Inquiry-based Teaching on Real Case

Xiaoli Ma[1, a*], Lili Zhang[1, b]

[1] School of Freshmen, Xi'an Technological University, Xi'an, 710021, China

[a]email:1450898954@qq.com, [b]email:1016874360@qq.com

Abstract. The Rule of total probability is an important formula in the probability theory and mathematical statistics, which provides an effective way to calculate complex probability problems in practical application, and often simplifies a complex probability calculation problem. However, in teaching, the application of the rule of total probability is a difficult point, because it is always difficult for students to master the analysis of complex problems, and they cannot divide the sample space well and find a suitable complete set of events, which makes it difficult to solve specific complex probability problems. This paper discusses a teaching design and executing process of the rule of total probability in probability and statistics course. This design fully demonstrates the student-centered principle. By introducing some interesting real cases the teacher proposes some questions, and then the students study the rule of total probability in a inquiry mode from the elementary to the profound. And to extend it which make use of the probability density function, and analyzing its extensive applications in insurance. This case-based inquiry teaching method, student-centered and case-driven, can fully mobilize students' learning interest and initiative, cultivate students' spirit of independent thinking and teamwork, and improve students' comprehensive application ability.

Keywords: Conditional probability; Rule of total probability; Probability density function; Inquiry-based teaching

1. Introduction

The rule of total probability is an important formula in probability theory, which provides an effective way to calculate complex probability and often simplifies a complex probability calculation problem. Many scholars have popularized the rule of total probability, which further expands the scope of use of the rule of total probability, and becomes an effective tool for us to solve more complex probability problems. However, it is found in teaching that there are always some problems when students use the rule of total probability to solve specific problems. Starting from the most basic conditional probability and total probability formula in probability theory, this paper guides students to explore the practical application of total probability formula in insurance from simple to deep. Start with a question for students to think about:

An auto insurance company classifies policyholders into general risks and unqualified risks according to certain classification procedures. Sixty percent were classified as medium risk. During the year, 2% of the average risks involved negligent accidents and 5% of the substandard risks involved negligent accidents. How many policyholders were involved in the at-fault?

To solve this problem, students need to be guided to explore the following new knowledge.

2. Conditional Probability and the Law of Total Probability

In practice, to calculate the probability of event A, we often need to attach a condition B to the event A to find its probability, this is the so-called conditional probability, and the following is to give its mathematical definition.

2.1 Definition of Conditional Probability

Let（Ω, F, P）be a probability space. Consider a random experiment with sample space Ω and events A and B. The conditional probability of event A given the information that event B Occurred is the number $P(A|B)$ given by

$$P(A \mid B)=\frac{P(AB)}{P(B)}.$$

The notation $A \mid B$ is used to represent event A under condition that D has occurred.

Where, Ω is a sample space, F is an algebra in, and P is a probability of an event.

It is said that the formula given above is a typical formula in probability theory, and in order to find the probability of complex events in practical problems, it is often possible to decompose it into the union of two or several incompatible simple events, find the probability of these simple events, and use the addition formula to find the probability of complex events. Then this method is generalized, is the rule of total probability, its basic idea is by introducing a variety of small premises, the sample space is properly decomposed into several parts, so that in each part (that is, under a variety of small premises) it is easy to obtain the desired probability.

2.2 Division of Sample Space

Let(Ω, F, P)be a probability space, $B_1, B_2, \cdots, B_i, \cdots$ is a pairings incompatible event column in F (finite or can be listed infinite), such that for everything i, there is $P(B_i)>0$, and $\bigcup_i B_i=\Omega$, then $B_1, B_2, \cdots, B_i, \cdots$ is called a partition or a complete family of events.

2.3 Rule of Total Probability

Suppose that Ω is a disjoint union of the events the event $B_1, B_2, \cdots, B_i, \cdots$ such that $B_i \cap B_j=\varnothing$ whenever $i \neq j$ and $B_1 \cup B_2 \cup \cdots \cup B_n=\Omega$, $P(B_i)>0$.

Then for any event A, and $A \subset \Omega$,

$$P(A)=\sum_{i=1}^{\infty} P(B_i) P(A|B_i). \tag{1}$$

Obviously, the rule of total probability gives a formula to calculate the probability of a complex event A. As long as the probability $P(D_i)$ of various causes B_i for the occurrence of A and conditional probability $P(A|B_i)$ of the occurrence of A under condition of the occurrence of various causes B_i are known, the probability of the occurrence of event A can be calculated using the rule of total probability.

2.4 Problem Solving

For the problem in the citation, students do not find a complete set of events B_1, B_2. The teacher guides students to analyze the previous examples and solve them using the rule of total probability.

Let B_1, B_2 and A represent the following event:

B_1: Policyholder classified as average risk.

B_2: Policyholder classified as substandard risk.

A: Policyholder involved in an at-fault accident.

Then, from the given information,

$$P(B_1)=0.60, \quad P(B_2)=0.40,$$

$$P(A \mid B_1)=0.02, \quad P(A \mid B_2)=0.05. \tag{2}$$

By the rule of total probability, the desired fraction is

$$P(A)=P(A|B_1) P(B_1)+P(A|B_2) P(B_2)=0.02 \times 0.60+0.05 \times 0.40=0.032. \tag{3}$$

In the application, it reminds students that when encountering complex events, they should pay attention not to miss known information, and quantify the information, find a complete set of events $B_1, B_2, \cdots, B_i, \cdots$ and use the rule of probability to solve.

3. Generalization and Application of the Rule of Total Probability

After learning the distribution of multidimensional random variables, students can be guided to generalize the total probability formula by using joint distribution, conditional distribution and edge distribution of random variables. The basic idea is to decompose an edge density into conditional density to simplify the problem to be solved.

3.1 Generalization of the Rule of Total Probability

Suppose that joint probability density function of two-dimensional random variables (X, Y) is $f_{X,Y}(x, y)$, the edge density functions are $f_X(x)$, $f_Y(y)$.Then its function of conditional density can be expressed by the following equation :

$$f_{X|Y=y}(x) = \frac{f_{X,Y}(x, y)}{f_Y(y)}, \tag{4}$$

$$f_{Y|X=x}(y) = \frac{f_{X,Y}(x, y)}{f_X(x)}. \tag{5}$$

In this way, the distribution form of the total probability formula can be obtained:

$$f_X(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x, s)ds = \int_{-\infty}^{+\infty} f_{X|Y=s}(x) f_Y(s)ds, \tag{6}$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f_{X,Y}(t, y)dt = \int_{-\infty}^{+\infty} f_{Y|X=t}(y) f_X(t)dt. \tag{7}$$

In the application, sometimes there will be mixed random variables, that is, one of them is discrete, the other is continuous, then you can use the distribution law.

Suppose that in two-dimensional random variables (X, Y), X is a continuous random variable, Y is a discrete random variable, and its distribution law is $p_Y(y)$, then

$$f_X(x) = \sum_y f_{X|Y=y}(x) p_Y(y), \tag{8}$$

If X is discrete and Y is continuous, then

$$p_X(x) = \int_{-\infty}^{+\infty} p_{X|Y=s}(x) f_Y(s)ds. \tag{9}$$

These formulas play an important role in solving problems with uncertain factors. In order to help students better understand the application of the generalized total probability formula, we explore the following case.

3.2 Application Case Analysis

A car insurer is trying to develop a model for claims size so that they can make good profits on their products. According to historical data, it has determined that the claim size distribution for policyholders classified as good risk has density

$$f_X(x) = 3e^{-3x}, x > 0. \tag{10}$$

And the claim size distribution for policyholders classified as good risk has density

$$f_X(x) = \frac{1}{2}e^{-\frac{x}{2}}, x > 0. \tag{11}$$

Where claims are measured in thousands of RMB. There is a 40% chance that a given policyholders is a bad risk. What is the probability that this policyholder\`s claim exceeds ¥2000?

By the case driven, students are encouraged to discuss the following questions in small groups:

- How to quantify the information of the case?

- How is the claim size expressed?

Then, teachers and students jointly explore and solve.

Let X be the claim size and let I be an indicator for being a bad risk. Then, from the given information,

$$I = \begin{cases} 1, & with\ \ probability\ \ 0.40, \\ 0, & with\ \ probability\ \ 0.60. \end{cases} \tag{12}$$

And further,

$$f_{X|I=0}(x) = 3e^{-3x}, x > 0, \tag{13}$$

$$f_{X|I=1}(x) = \frac{1}{2}e^{\frac{-x}{2}},\ \ x > 0. \tag{14}$$

Hence, by the Generalization of the rule of total probability, the density function for X is given by

$$\begin{aligned} f_X(x) &= f_{X|I=0}(x)p_I(0) + f_{X|I=1}(x)p_I(1) \\ &= 3e^{-3x} \times 0.60 + \frac{1}{2}e^{-\frac{x}{2}} \times 0.40 \\ &= 1.8e^{-3x} + 0.2e^{-\frac{x}{2}}. \end{aligned} \tag{15}$$

Thus, the required probability is

$$\begin{aligned} P(X > 2) &= \int_2^{+\infty} f_X(x)dx \\ &= 1.8 \times \int_2^{\infty} e^{-3x}dx + 0.2 \times \int_2^{\infty} e^{-\frac{x}{2}}dx \\ &= 0.6e^{-6} + 0.4e^{-1}. \end{aligned} \tag{16}$$

That is, the probability that the claim size is greater than one unit is $0.6e^{-6} + 0.4e^{-1}$.

In this problem, the key is to ask for the density function of the claim amount under different risks, so we must quantify the information number of the problem, set an indicator variable, so that the problem becomes easier to solve.

4. Conclusions

The generalized the rule of total probability can be used to solve problems with multiple different types of random variables, and can also be solved by using the distribution function of random variables. In fact, many knowledge points of probability theory can be used to solve a series of uncertain problems in investment, insurance, engineering, such as Bayes' theorem, mixed probability distribution and so on.

Through this case driven and inquiry teaching method inspired students to realize that the key in this problem is to ask the density function of the claim size under different risks, so we must quantify the information set up by the problem, set an indicator variable, so that the problem becomes easier to solve. This student-centered and inquiry-based teaching method can improve students' application ability and innovation ability to solve practical problems.

This teaching design is student-centered and fully emphasizes the principal position of students in learning. In the whole learning process, problems are elicited, analyzed and discussed through practical cases that students are interested in, and teachers guide and help students to think independently. This design significantly improves students' learning interest and reduces the difficulty of learning. Effectively help students understand the meaning of the total probability formula and master its application, and students also show high enthusiasm and participation in the actual teaching process. Therefore, this design may be a more suitable learning method for students, and to a certain extent, it also meets the requirements for the training of application-oriented innovative talents under the background of new engineering. It has certain reference value and can be tried in practical teaching.

Acknowledgements

This work is supported by Xi’an Technological University teaching reform project (21JGG001).

References

[1] Zhou Sheng, Shiqian Xie, Chengyi Pan, Probability Theory and Mathematical Statistics [M], Beijing: Higher Education Press, 2019.

[2] Michael A. Bean, Probability: The Science of Uncertainty with Applications to Investment, Insurance, and Engineering [M], Beijing: China Machine press, 2003.

[3] Xiaoli Ma, Liang Zhang, Extension of Law of Total Probability with Applications in Insurance [J], Advanced mathematics research, 2010.1:70-71.

[4] Caihong Han, Ying Xia, Simin Pang, et al, Thought and politics in probability theory and mathematical statistics, Exploration and fusion of elements [J], Educational Observation, 2019, 9(1):103-105.

[5] Hui Wang, Ling Yuan, Hongju Li. New teaching of Probability Theory and Mathematical Statistics Exploration of model and teaching means [J], Journal of Huangshan University, 2019, 21(5):102-105.

[6] Haixia Zhang, "Probability Theory and Mathematical Statistics" Based on Innovation Ability Training Teaching Reform and Exploration [J], Science and Technology Economics Guide, 2019, 28(21):115-116.

[7] Xinhong Liu, Yuan Feng, Dexian Qiao, Student-Centered Teaching Practice of Advanced Mathematics [J], University Mathematics, 2019, 36(3):45-50.

[8] Zhumei Liang, Yinshan Qi, Yuhao Cen, Construction of "Student-centered" teaching style Scale: Reliability and Validity Test and its application [J], China University Teaching and Learning, 2020(11):73-80.

[9] Jing Wang, Fang Xiaofeng, Yingqi Li, Design and practice of "learning-centered" high is onumeracy teaching on line [J], University Mathematics, 2021, 36 (6):101-109.

[10] Kaining Wu, Xuechen Wang, Liming Wang, Thinking on the Teaching Mode of College Mathematics Courses: the form of "Rain Classroom" [J], University Mathematics, 2021, 37 (l):27-32.

[11] Mengqiao Li, Research and Practice of Advanced Mathematics Teaching based on rain classroom and Group Classroom [J], Curriculum Teaching and Learning, 2020 (9):127-128.

[12] Mengxia Li, Discussion on the teaching design of "Advanced Mathematics" course in rainy class [J], Curriculum Teaching, 2020, (33):130-131.